Complete program

(Room 1C150, 1st floor)

Monday, 25/09/2017

- 8.15-9.00 **Registration**
- 9.00-9.30 **Opening**
- 9.30-10.15 **R. B. Vinter** - *A Unified Approach to State Constrained Optimal Control, Based on Distance Estimates*
- 10.15-11.00 **M. Monteiro Marques** - *State-dependent sweeping processes*

11.00 coffe break

- 11.30-12.15 **B. Mordukhovich** - *Method of discrete approximations for controlled sweeping processes and their applications*
- 12.15-13.00 **M. Negri** - *Rate-independent unilateral evolutions \break for Ambrosio-Tortorelli functionals and applications*

13.00 Lunch

- 14.30-15.15 **A. Bressan**- *Models of constrained biological growth*
- 15.15-16.00 **F. Santambrogio** - *Evolution PDEs under density constraints*

16.00 coffe break

- 16.30-17.15 **A. Agrachev** - *Switching in time-optimal problems*
- 17.15-18.00 **E. Trélat** - *Positive minimal time for the control of state constrained dynamical systems*

Tuesday, 26/09/2017

- 9.00-9.40 **S. Migórski** *Dynamic History--Dependent Variational--Hemivariational Inequalities with Applications*
- 9.40-10.20 **D. Tiba** - *Some Remarks on State Constraints and Mixed Constraints*
- 10.20-11.00 **M. Sofonea** -*Inequality Problems, Sweeping Precesses and Optimal Control in Contact Mechanics*

11.00 coffe break

- 11.30-12.10 **P. Krejčí** - *Optimal piezoelectric energy harvesting strategy*
- 12.10-12.50 **G. Wachsmuth** - *Optimal control of a rate-independent evolution equation via viscous regularization*

12.50 lunch

- 14.50-15.30 **M. Brokate** - *Differential sensitivity in rate independent problems*
- 15.30-16.10 **O. Makarenkov** -*Stabilization of quasistatic evolution of elastoplastic systems subject to periodic loading*

16.10 coffe break

- 16.40-17.00 **M. Palladino** -*Sweeping Process and Optimal Control*
- 17.00-17.20 **Ch. E. Arroud** - *A Mayer Problem for A Controlled Sweeping Process*

- 17.20-17.40 **Tan Cao** -*Optimal Control for a Controlled Sweeping Process with Applications to the Crowd Motion Model*
- 17.40-18.00 **A. Vieira** -*Optimal Control of Linear Complementarity Systems*

Wednesday, 27/09/2017

- 9.00-9.40 **P. Cannarsa**- *Mean Field Games with State Constraints*
- 9.40-10.20 **A. Arutyunov**- *Investigation of Optimality Conditions in Control Problems with State Constraints*
- 10.20-11.00 **L. Grüne** - *The role of state constraints for turnpike behaviour and strict dissipativity of optimal control problems*

11.00 coffe break

- 11.30-12.10 **P. Wolenski** - *The method of characteristics in Fully Convex Control problems with state constraints*
- 12.10-12.50 **A. Marigonda** - *Mayer and minimum time problem for multi-agent systems*

12.50 lunch

- 14.50-15.30 **M. Mazzola** - *Neighboring feasible trajectories in infinite dimension*
- 15.30-16.10 **E. Marchini** -*Infinite dimensional state constrained optimal control problems*
- 16.10-16.50 **H. Zidani** -*Value Function and Optimal Trajectories for some State-Constrained Control Problems*

16.50 coffe break

- 17.00 - *Informal Meeting on EU Projects*

Social Dinner (Ristorante "Isola di Caprera", 8.00 p.m.)

Thursday, 28/09/2017

- 9.00-9.40 **A. Vladimirov** *Derivatives of sweeping processes as continuous products of matrices*
- 9.40-10.20 **V. Recupero** - *BV-norm continuity of sweeping processes \\ driven by a set with constant shape*
- 10.20-11.00 **J. Venel** - *Differential inclusions and applications*

11.00 coffe break

- 11.30-12.10 **R. De Pinho** – *Optimal Control Problems with Mixed Constraints: Special Applications*
- 12.10-12.50 **F. Bagagiolo** - *Optimal control with several targets: the need of a rate-independent memory*

12.50 lunch

- 14.50-15.10 **P. Gidoni** - *An introduction to rate-independent soft crawlers*
- 15.10-15.30 **M. Zoppello** - *Controllability properties of dynamical systems with hysteresis*
- 15.30-15.50 **T. Haddad** -*On Evolution Equations Having Hypomonotonicities of Opposite Sign*

- 15.50-16.10 **E. Vilches** -*Galerkin-Like method for generalized perturbed sweeping process with nonregular sets*

16.10 coffe break

- 16.40-17.00 **A. Lai** - *Global asymptotic controllability for unbounded control systems*
- 17.00-17.20 **S. Cacace** -*PacMan^{HJ}: a classical arcade videogame powered by Hamilton-Jacobi equations*

Friday, 29/09/2017

- 9.00-9.40 **C. Mariconda** – *One-dimensional non-autonomous integral functionals with discontinuous non-convex integrands: Lipschitz regularity and DuBois-Reymond necessary conditions*
- 9.40-10.20 **P. Bettiol** - *Differential Games confined to path-wise constraints*
- 10.20-10.40 **S.Aronna** -*Second order optimality conditions of control-affine problems with scalar state constraint*
- 10.40-11.00 **N. Khalil** -*Necessary Optimality Conditions For Average Cost Minimization*
- *Problems*

11.00 coffe break

- 11.30-12.10 **M. Motta** - *Normality and Gap Phenomena in Optimal Impulsive Control*
- 12.10-12.20 **Conclusion**

12.20 lunch

Switching in time-optimal problems

A. Agrachev and Carolina Biolo

SISSA, Trieste, Italy

We study the time-optimal problem for generic control-affine system of the form:

$$\dot{x} = f_0(x) + \sum_{i=1}^{k} u_i f_i(x), \quad x \in R^n, \quad \sum_{i=1}^{k} u_i^2 \leq 1, \quad 1 < k < n,$$

and try to decode the structure of jump discontinuities of the optimal control in terms of Lie bracket relations between the vector fields $f_0, f_1, \dots f_k$. Pontryagin Maximum Principle, the blow-up procedure, and partially hyperbolic dynamics allow to reduce the problem to the study of a simple dynamical system on the sphere S^{k-1}. This is a joint work with Carolina Biolo, SISSA.

Second order optimality conditions of control-affine problems with scalar state constraint

M. Soledad Aronna and J. Frédéric Bonnans and Bean-San Goh

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Keywords : optimal control, control-affine problem, second order condition, state constraint, singular arc

We establish second order necessary and sufficient optimality conditions for a class of control-affine problems with a scalar control subject to bounds and a scalar state constraint, i.e., we consider the system

$$\dot{x}(t) = f_0(x(t)) + u(t)f_1(x(t)),$$

with the constraints

$$u_{\min} \leq u(t) \leq u_{\max}, \qquad g(x(t)) \leq 0,$$

and the initial-final state constraints

$$\Phi(x(0), x(T)) \in \{0\}_{\mathbb{R}^{n_1}} \times \mathbb{R}^{n_2}_{-}.$$

We apply the Goh transform to write the second variation in a new space of perturbations in which it might be coercive. We show that this coercivity implies local Pontryagin optimality.

We show an example to illustrate our results, and we show how the shooting algorithm can be employed in this state-constrained framework.

A Mayer Problem for A Controlled Sweeping Process

Chems Eddine Arroud and Giovanni Colombo

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Keywords : Moreau's sweeping process, adjoint equation, Optimal control, Pontryagin Maximum Principle.

We Consider the free endpoint Mayer problem for the controlled Moreau process ;
Minimize $h(x(T))$ subject to

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t))), \\ x(0) = x_0 \in C(0)\ , \quad u(t) \in U \end{cases} \tag{0.1}$$

where $C(t)$ is a closed non-convex moving set, with normal cone $N_{C(t)}(x)$ at $x \in C(t)$, U being the control set and f being smooth.
We prove necessary optimality conditions in the form of Pontryagin maximum principle. We combine techniques from [2] and from [1], a kind of inward/outward pointing condition is assumed on the reference optimal trajectory at the times where the boundary of $C(t)$ is touched.
The adjoint equation is not the classical one, because a signed vector measure appears, and also the maximality condition is different.

[1] M. Brokate and P. Krejčí, *Optimal control of ODE systems involving a rate independent variational inequality*, Discrete and continous dynamical systems serie B. Volume 18 (2013), 331-348.
[2] M. Sene, L. Thibault, *Regularization of dynamical systems associated with prox-regular moving sets*, Journal of Nonlinear and Convex Analysis 15 (2014), 647-663.
[3] Ch. Arroud and G. Colombo. A Maximum Principle for the controlled Sweeping Process. Set-Valued Var. Anal (2017), 1-23.

Investigation of Optimality Conditions in Control Problems with State Constraints

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Keywords : optimal control, state constraints, non-degenerate maximum principle

This talk concerns the study of optimal control problem with state constraints. After the problem formulation we consider the necessary optimality conditions, or the maximum principle, in the Dubovitskii-Milyutin form. As is well-known, these conditions may degenerate. Therefore, after the formulation of the maximum principle we study various conditions for non-degeneracy, including the controllability and point-wise controllability hypotheses. Then, we pass to the Gamkrelidze form of the maximum principle which is equivalent to the form given by Dubovitskii and Milyutin under the assumption that the second-order derivative of the state constraint function exists. The non-degenerate maximum principle is reformulated in the framework of the Gamkrelidze approach as well. Then, some remarks are given regarding the conditions for continuity of the measure-multiplier and regarding some further generalization.

Optimal control with several targets: the need of a rate-independent memory

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Keywords : Minimum time, visiting problem, Hamilton-Jacobi equations, memory, hysteresis, mean field games

Consider a minimum time problem where the goal is to touch several targets (closed subsets of the state-space R^n) in the possible minimal time: a so-called optimal visiting problem. When the targets are more than one, then the dynamic programming principle does not hold, because the actual state variable $x \in R^n$ does not encode the information whether the i-th target has been already reached or not: it is without memory. Hence we have to add some further variables, one per each target, whose time-dependent values bring all the needed information: they must have memory. Obviously, such a memory must be rate-independent, that is depending only on the sequence of values reached by the state and not on the time scale; in other words it must exhibits hysteresis ([1], [2]). The problem then becomes an optimal control problem with hysteresis in the augmented state space. The goal is to study the corresponding Hamilton-Jacobi equation in the framework of the viscosity solutions theory ([3]). Starting from an old result [4] concerning dynamic programming principle and Hamilton-Jacobi equations for optimal control problems with hysteresis, I will move to more recent results [5] for the optimal visiting problem with the added memory variables. Then I will sketch some extensions of [5] and a recent related study, in the framework of the mean field games theory, concerning the control of the tourists flow in a historic city center [6].

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[2] A. Visintin: *Differential Models of Hysteresis*, Springer, 1994.
[3] M. Bardi, I. Capuzzo Dolcetta: *Otpimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*, Birkh́auser, 1997.
[4] F. Bagagiolo: Dynamic programming for some optimal control problems with hysteresis, NoDEA Nonlinear Differential Equations Appl., 2002.
[5] F. Bagagiolo, M. Benetton: *About an optimal visiting problem*, Appl. Math. Optim., 2012.

[6] F. Bagagiolo, R. Pesenti: *Non-memoryless pedestrian flow in a crowded environment with target sets*, Preprint 2017.

Differential Games confined to path-wise constraints

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Keywords : Differential Games, state constraints

Differential Games concern the balance of "optimal" strategies applied by two opposing players, who have conflicting notions of "best" performance of the dynamical system which they are both trying to control. Typically we are given an information pattern for the two players prescribing that each of them choose his own control at each instant of time without knowing the future choices of the opponent. This can be made rigorous by introducing the notion of *non-anticipative strategy*, establishing that each player knows the past and present behaviour of the other player. When a state constraint is imposed a major difficulty is represented by the construction of admissible controls and strategies for the two players. We shall discuss some recent results which allow to derive regularity properties of the value function in state constrained Differential Games and to justify interpreting it as a (possibly unique) generalized solution of the Hamilton-Jacobi-Isaacs equation.

Models of constrained biological growth

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Keywords : Stem growth, evolution equations with constraints

We consider a PDE model for the growth of a plant stem, accounting for linear elongation, gravity, and the presence of external obstacles such as branches of other plants. Due to the one-sided constraints, this can be reformulated as a differential inclusion within a suitable H^2 space. Existence and continuous dependence of solutions is proved, up to the first time when a specific "breakdown configuration" is reached. Asymptotic stability in the upward direction will also be discussed.

Differential sensitivity in rate independent problems

Martin Brokate

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Keywords : hysteresis, sensitivity, weak derivatives, optimality conditions

Rate independent problems are inherently nonsmooth. We discuss the existence and form of weak derivatives, in particular Newton derivatives, of hysteresis operators as well as control problems where hysteresis operators appear.

PacMan$^{\mathbf{HJ}}$: a classical arcade videogame powered by Hamilton-Jacobi equations

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Keywords : Differential Games, Dynamic Programming Principle, Hamilton-Jacobi Equations

PacMan is maybe the most famous arcade videogame of the 80's, widely played in the last decades and still appreciated nowadays. PacMan is a yellow creature shaped like a pizza slice, confined in a maze covered by pills and haunted by ghosts. The game is endless and the ultimate goal of the player is just to let PacMan survive as much as possible, avoiding the ghosts and eating all the pills to earn points and move to the next level. The only help are some special pills that make the ghosts temporarily harmless.

In this talk I revisit this legendary videogame in the framework of Pursuit-Evasion differential games, leading to a far from trivial mathematical problem. The celebrated dynamic programming principle allows to recast the basic game in terms of a static Hamilton-Jacobi-Isaacs equation. Its solution can be efficiently pre-computed by a simple fixed point algorithm based on a semi-Lagrangian scheme, from which optimal strategies for both PacMan and the ghosts can be readily synthetized. Using the graph structure of the maze and some mild assumptions on the system, it is possible to drastically reduce the dimension of the problem, embedding both state and control constraints in a simple data structure and working only with integers in exact arithmetic. I will also discuss how the intermediate problem of the special pills can be set in the framework of hybrid controlled systems. In this case a solution can be found solving a static Hamilton-Jacobi-Isaacs equation on a hierarchical varifold, obtained by duplicating the state space across the special pills locations. Finally, I will spend some words on the full problem including the normal pills, which is still open and under investigation.

Mean Field Games with State Constraints

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This talk will address deterministic mean field games for which agents are restricted in a closed domain of $\mathbf{R}^n$ with smooth boundary. In this case, the existence, uniqueness, and regularity of Nash equilibria cannot be deduced as for unrestricted state space because, for a large set of initial conditions, the uniqueness of solutions to the minimization problem which is solved by each agent is no longer guaranteed.

We will therefore attack the problem by considering a relaxed version of it, for which the existence of equilibria can be proved by set-valued fixed point arguments. We will then give a uniqueness result for such equilibria under a classical monotonicity assumption. Finally, we will analyze the regularity of the relaxed solution and show that it satisfies the typical first order mean field games system.

Optimal Control for a Controlled Sweeping Process with Applications to the Crowd Motion Model

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Keywords : Controlled sweeping process, Hysteresis, Varational analysis, Discrete approximations, Generalized differentiation, Necessary optimality conditions, Crowd motion model.

This talk concerns optimal control problems for a new class of dynamics systems governed by the (Moreau) sweeping process which arises in various problems of hysteresis, ferromagnetism, electric circuits, phase transitions, economics, etc. The dynamics of such system was introduced in the 1970's by J. J. Moreau to model quasi-static evolution processes subject to unilateral constraints, and it can be described by the normal cone mapping to moving polyhedral convex sets. The main attention is paid to deriving necessary optimality conditions for optimal control problems using the method of discrete approximations. It should be emphasized that the velocity mapping (described by the normal cone) in the differential inclusion is highly non-Lipschitz and unbounded, which cannot be treated by means of known results in optimal control for differential inclusions. Such challenging issues can be overcome by developing the method of discrete approximations married with appropriate generalized differential tools of modern variational analysis. We also discuss new applications to the controlled crowd motion model of traffic equilibria.

This talk is based on the joint research with Boris Mordukhovich (Wayne State University, Detroit, MI, USA.)

An introduction to rate-independent soft crawlers

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Keywords : crawling locomotion, anisotropic dry friction, rate-independent systems, sweeping processes

The inclusion of elastic components in the modelling and design of biomimetic crawlers endows these systems with new compliance capabilities, but at the same time rises additional challenges to the analysis of their locomotion properties. The mathematical theory of rate-independent systems and sweeping processes provides an effective framework to address these issues. For instance, applying such techniques, the motility of the systems can be analysed more readily through the introduction of *stasis domains*, while characteristics and effects of other associated phenomena, as friction anisotropy, can be emphasized.

The aim of this talk is to provide, with the aid of representative toy models, an essential introduction to the modelling of rate-independent soft crawlers, highlighting the mathematical structure involved and presenting the main features and challenges.

The role of state constraints for turnpike behaviour and strict dissipativity of optimal control problems

Lars Grüne and Roberto Guglielmi

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Keywords : finite and infinite horizon optimal control, optimal equilibria, model predictive control

In recent years, the turnpike property has received renewed interest, particularly in model predictive control and in optimal control for PDEs. The turnpike property describes a particular behaviour of optimal or near optimal trajectories in both finite and infinite horizon optimal control, which has been extensively studied in mathematical economy since the 1940s. The property demands that these trajectories most of the time stay close to a particular reference trajectory which is often constant, i.e., an equilibrium. The turnpike property opens the way to efficient numerical methods for computing approximately optimal trajectories on long horizons, which we will illustrate in the first part of this talk.

The crucial question is then to determine which optimal control problems have the turnpike property. To this end, the concept of dissipativity, in the systems theoretic sense introduced by Willems in the early 1970s, turns out to be useful. Particularly, under suitable controllability conditions a property called strict dissipativity is equivalent to the occurrence of the turnpike property for all near optimal trajectories.

In this talk these relations will first be explained and then conditions under which strict dissipativity and the turnpike property hold will be discussed. Particularly, we will explain that (and why) already in the simplest possible setting, i.e., for linear quadratic optimal control problems, the conditions on the optimal control problem become considerably less restrictive in the presence of state constraints.

On Evolution Equations Having Hypomonotonicities of Opposite Sign

Tahar Haddad and Chems Eddine Arroud

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Keywords : functional differential inclusion; normal cone; sweeping process; perturbation

The purpose of the present talk is to study a Cauchy problem of the form

$$\dot{x}(t) \in -N_C(x(t)) + F(x(t)), \quad F(x) \subset \partial g(x), \quad x(0) = x_0 \in C, \qquad (0.2)$$

where x is in a infinite dimensional Hilbert space H, the closed set C is locally prox-regular at x_0 (hence N_C is hypomonotone set valued mapping), g is a primal lower nice function (shortly pln) and the set valued mapping F is upper semicontinuous with compact values defined over some neighborhood of x_0. Hence, the set valued mapping F is not necessarily the whole subdifferential of g , and we take the plus sign, instead of the classical minus. We prove (local) existence of solutions.
In [2], the anthors have considered the problem (0.2), where F is upper semicontinous, not necessarily convex set-valued mapping contained in the subdifferential of a Lipschitz convex function g . Moreover the closed set C was supposed to be compact and uniformly r-prox-regular.
It is also worth mentioning that problem (0.2) in the more general form

$$\dot{x}(t) \in -\partial f(x(t)) + F(x(t)), \quad F(x) \subset \partial g(x), \quad x(0) = x_0 \in dom f, \quad (0.3)$$

with g is φ-convex of order two and f has a φ-monotone subdifferential of order two (shortly $f \in MS(2)$) has been studied in [1], notice that we can obtain (0.2) from (0.3), by taking $f = \delta_C$, the indicator function of the set C, i.e. $\delta_C(x) = 0$ for $X \in C$ and $\delta_C(x) = \infty$ for $x \in H \setminus C$. Indeed, as C is locally pox-reqular, then $f = \delta_C$ is pln (proposition 3.31 [3]) and so $MS(2)$.
Despite the similarity of (0.2) and (0.3), the problems are quite different, since in general with $f = \delta_C$ the level set $\{x \in H; f(x) \leq r\}$ is compact,

and these were basic assumption in [1]. Since these condition do not holds for (0.2, she has to be replaced by suitable substitute in case of sweeping processes (0.2).

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Necessary Optimality Conditions For Average Cost Minimization Problems

Nathalie Khalil and Piernicola Bettiol

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Keywords : control system, uncertainties, average cost, necessary conditions

Parameter-dependent control systems appear a natural framework for applications in which the model design has to take into account various uncertainties. In these circumstances the performance criterion can be given in terms of an average cost. In this talk, we describe a class of optimal control problems in which unknown parameters intervene in the data, and we provide its necessary optimality conditions.

Optimal piezoelectric energy harvesting strategy

Barbara Kaltenbacher and Pavel Krejčí

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Keywords : Piezoelectricity, hysteresis, optimal control

Magneto-mechanical or electro-mechanical coupling in magnetostrictive or piezoelectric materials exhibits strong hysteresis effects, see [4]. In order to describe the self-similar character of both the magnetization loops and the "butterfly" magnetostrictive loops, a model based on the Preisach hysteresis operator and its energy potential operator acting on an auxiliary self-similar variable was proposed in [2]. An energy harvesting problem for a periodically vibrating magnetostrictive element was considered in [3]. Well-posedness of a PDE problem with an analogous model for piezoelectric materials was established in [5].

This contribution is based on the results of [6], where we study the harvesting problem with an extended model for piezoelectricity, which takes into account also the feedback effects. Thus, it is able to explain the phenomenon of mechanical depolarization which was neglected in the previous models. We assume that the process is governed by the equations

$$\begin{aligned} \rho\ddot{\varepsilon} + \nu\dot{\varepsilon} + c\varepsilon - eE + f'(\varepsilon)\,\mathcal{V}[u] + \tfrac{1}{2}b'(\varepsilon)\mathcal{P}^2[u] &= \sigma, \\ \tfrac{\mathrm{d}}{\mathrm{d}t}\Big(e\varepsilon + \kappa E + \mathcal{P}[u]\Big) + \alpha E &= 0, \end{aligned}$$

where ε (the strain) and E (the electric field) are the unknowns of the problem, u given by the implicit formula $u = \frac{1}{f(\varepsilon)}(E - b(\varepsilon)\mathcal{P}[u])$ is the self-similar variable, $\mathcal{P}$ is a Preisach operator, $\mathcal{V}$ is its potential, $f(\varepsilon)$ is the self-similarity function, $b(\varepsilon)$ is the feedback function, $\sigma = \sigma(t)$ is a given applied force, and $\rho, \nu, c, e, \kappa, \alpha$ are constant physical coefficients. The goal is to maximize a cost functional representing the harvested energy over a given time interval $[0, T]$ with respect to the mass and the resistance of the harvester. We prove that the optimization problem has a solution, and in the special case that the Preisach operator can be locally represented by the play operator, we derive the necessary optimality conditions.

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[4] M. Kamlah: Ferroelectric and ferroelastic piezoceramics modeling of electromechanical hysteresis phenomena. *Continuum Mechanics and Thermodynamics* **13** (2001), 219–268.
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[6] B. Kaltenbacher, P. Krejčí: Energy harvesting with a piezoelectric device vibrating in thickness direction. To appear.

Global asymptotic controllability for unbounded control systems

Anna Chiara Lai, Monica Motta and Franco Rampazzo

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Keywords : Optimal control, asymptotic controllability, exit-time problems, unbounded controls, vector polynomials.

We present some results on exit-time minimum problems with non-negative running cost and with unbounded controls.

We consider a particular Lyapunov function, the Minimum Restraint Function. We show that, under mild assumptions on the dependence of the data on inputs, its existence provides global asymptotic controllability of the system and a state-dependent upper bound for the infima.

We then discuss the particular case of systems with a polynomial dependence on controls. This class of systems include control-quadratic systems that are suitable to model mechanical systems controlled via time-varying, holonomic constrains. We show that the algebraic and convexity properties of polynomial system allow for some simplified versions of the main result.

Stabilization of quasistatic evolution of elastoplastic systems subject to periodic loading

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Keywords : Elastoplastic springs, Moreau sweeping process, Quasistatic evolution, Periodic loading, Stabilization

This talk discusses an analytic framework to design both stress and stretching/compressing T-periodic loadings which make the quasi-static evolution of a parallel network of elastoplastic springs converging to a unique periodic regime. The solution of such an evolution problem is a function $t \mapsto (e(t), p(t))$, where $e_i(t)$ and $p_i(t)$ are the elastic and plastic deformations of spring i, uniquely defined on $[t_0, \infty)$ by the initial condition $(e(t_0), p(t_0))$. After we rigorously convert the problem into a Moreau sweeping process with a moving polyhedron, it becomes natural to expect (based on a result by Krejci) that the solution $t \mapsto (e(t), p(t))$ always converges to a T-periodic function. The achievement of this work is in spotting a class of sweeping processes and closed-form estimates on eligible loadings where the Krejci's limit doesn't depend on the initial condition $(e(t_0), p(t_0))$ and so all the trajectories approach the same T-periodic solution. The proposed class of sweeping processes is the one whose moving polyhedron $C(t)$ doesn't have pairwise parallel facets. We further link this geometric condition to mechanical properties of the given network of springs. We discover that the facets of $C(t)$ are never pairwise parallel, if the number of stretching/compressing constraints is 2 less the number of nodes of the given network of springs and when the magnitude of the stress loading is sufficiently large (but admissible). In other words, we offer an analogue of the high-gain control method for elastoplastic systems, which can be used e.g. to design the properties of smart materials.

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Infinite dimensional state constrained optimal control problems

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Keywords : infinite dimensional optimal control, value function, necessary conditions, normal maximum principle

We study infinite dimensional optimal control problems under state constraints. The control system analyzed is the following:

$$\dot{x}(t) = Ax(t) + f(t, x(t), u(t)), \tag{0.4}$$

with $x(t) \in K$, for $t \in [0, 1]$. Among the solutions of (0.4) we aim to minimize the functional

$$g(x(1)).$$

The setting is quite general, hence our analysis applies to some interesting and delicate frameworks: the operator A is the infinitesimal generator of a strongly continuous semigroup $S(t) : X \to X$, and X is an infinite dimensional separable Banach space.

Assuming inward pointing conditions on the set of constraints K, we propose results on regularity of the value function associated to the above optimal control problem, together with results on variational inclusions allowing to prove first order necessary optimality conditions. Further, normality of the maximum principle is discussed.

Our analysis will be applied to models involving different kind PDEs.

One-dimensional non-autonomous integral functionals with discontinuous non-convex integrands: Lipschitz regularity and DuBois-Reymond necessary conditions

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Keywords : Calculus of Variations, Maximum Principle, Nonsmooth analysis, Lipschitz regularity

We consider the Lipschitz regularity of the minimizers to

$$\int_a^b L(t, x(t), x'(t))\, dt,\ x \in W^{1,1}([a,b]),$$

$$x(a) = A, x(b) = B, x(t) \in \Sigma \subset R^n.$$

A celebrated result by F. Clarke and R. Vinter asserts the Lipschitz regularity of the minimizers to a superlinear autonomous one-dimensional Lagrange problem of the calculus of variations when the Lagrangian is local Lipschitz and convex in the velocity variable. G. Dal Maso and H. Frankowska formulated the result when the integrand is just Borel.

In a joint paper with P. Bettiol we extend the result to a wide class of non-autonomous Lagrangeans. Lispchitzianeity of the minimizers is obtained under a weaker growth condition than superlinearity, and a necessary DuBois-Raymond type necessary condition is deduced. Our methods involve Clarke's formulations of the maximum principle.

Mayer and minimum time problem for multi-agent systems

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Keywords : multi-agent systems, differential inclusions, optimal control problem

Multi-agent systems are dynamical systems involving a huge number of independent agents, for which only a statistical description is available. These systems can be modeled by (time-dependent) measures, where the measure of a set B yields the fraction of agents contained in B. We will consider some optimal control problems in this framework, proving a dynamic programming principle for the value function V which leads to an Hamilton-Jacobi-Bellman equation in the space of probaiblity measure, solved in a suitable viscosity sense by V. In some special cases, we can also prove an uniqueness result for such an equation.

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[2] G. Cavagnari, A. Marigonda and B. Piccoli: Averaged time-optimal control problem in the space of positive Borel measures, to appear in *ESAIM: Control, Optimisation and Calculus of Variations*.

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Neighboring feasible trajectories in infinite dimension

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Keywords : Semilinear differential inclusion, state constraints, neighboring feasible trajectory theorem

In control theory under state constraints, approximations of solutions of the control system by trajectories lying in the given constraint set are very useful for applications. For instance they can be used in the study of the regularity of the value function and the non degeneracy of first order necessary conditions for optimality. In this talk we are interested in this kind of results when dealing with differential inclusions of the form

$$\dot{x}(t) \in Ax(t) + F(t, x(t)),$$

with $x(\cdot)$ staying in a given closed subset of an infinite dimensional separable Banach space. Here, the operator A is the infinitesimal generator of a strongly continuous semigroup. In her talk, Elsa Maria Marchini will show how it is possible to apply this result in order to prove the Pontryagin maximum principle and its normality for constrained optimal control problems involving PDEs.

Dynamic History–Dependent Variational–Hemivariational Inequalities with Applications

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Keywords : Variational inequality; hemivariational inequality; viscoelastic; frictional contact

We provide recent results on existence and uniqueness for a first order variational-hemivariational inequality with history-dependent operators. The history-dependent operators appear in both a locally Lipschitz nonconvex superpotential and in a convex potential. The results are applied to a dynamic frictional viscoelastic contact problem with multivalued nonmonotone subdifferential boundary conditions.

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State-dependent sweeping processes

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Keywords : state-dependent evolution problems convex prox-regular sets

A basic state-dependent sweeping process in a Hilbert space may be written in short as

$$-\frac{du}{dt}(t) \in N_{C(t,u(t))}(u(t)),$$

where $u : I = [0,T] \to H$ is an absolutely continuous function, $C(t,u)$ are subsets of H and $N_{C(t,u)}(x)$ denotes the outward normal cone to $C(t,u)$ at x. The above differential inclusion requires that $u(t) \in C(t,u(t))$, for all t, including for the initial value. The r.h.s. may also contain standard $f = f(t,u)$ terms.

Basic results for this problem will be presented, along the lines of the papers [1]-[4]. In their simplest form [1], the sets $C(t,u)$ are closed and convex and the dependence $(t,u) \mapsto C(t,u)$ is Lipschitz-continuous w.r.t. Hausdorff distance h

$$h(C(t,u), C(s,v)) \leq L_1 |t-s| + L_2 |u-v|_H,$$

with $L_2 < 1$. In infinite-dimensional settings, compactness assumptions may be added, for technical reasons. An example of application is given in [2].

Generalizations may be obtained when the sets are not far from being convex, say prox-regular or phi-convex, as in [3]. It is also possible to work in ordered Hilbert spaces [4].

If the normal cone to $C(t,u)$ is replaced by a maximal monotone operator $A(t,u)$, the problem is to find more generally $u : I \to H$ such that

$$-\frac{du}{dt}(t) \in A(t,u(t))(u(t))$$

meaning that $u(t) \in D(A(t,u(t)))$ and that, for all $v \in D(A(t,u(t)))$ and $z \in A(t,u(t))v$, one has

$$\langle \frac{du}{dt}(t) + z, v - u(t) \rangle \geq 0.$$

Assuming that the dependence of the m.m.o. on the state is measured by Vladimirov's pseudo-distance, one extends the previous study.

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Normality and Gap Phenomena in Optimal Impulsive Control

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Keywords : Optimal impulsive control, Necessary conditions, Lavrentiev-type gap phenomena

For some optimal control problems arising in mechanics and other areas, existence of minimizers can be guaranteed if the optimal control problem is posed, not over the conventional class of absolutely continuous trajectories, but over some larger class of impulsive trajectories: the latter can be shortly described as the set of the C^0 limits of the graphs of absolutely continuous trajectories corresponding to L^1 controls [1]. In the presence of end-point constraints, the infimum cost for the extend problem may happen to be strictly less than the infimum cost. Similarly to what some authors have done for relaxation of optimal control problems [2], [3], [4], we aim at identifying conditions that avoid the occurrence of such a gap, which would make the domain's extension useless to study the original problem. We state an Extended Pontrjagin Maximum Principle and prove that normality guarantees the absence of infimum gaps. Successively, we identify some verifiable sufficient conditions on the data of the problem which imply normality of each minimizer.

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Method of discrete approximations for controlled sweeping processes and their applications

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This talk concerns optimal control problems for several versions of the controlled sweeping process governed by dissipative differential inclusions. Such dynamic optimization problems constitute a new and challenging class in optimal control of discontinuous systems with intrinsic state constraints of inequality and equality types. We develop the method of discrete approximations for this class of optimal control problems and establish its strong convergence properties. Using advanced tools of variational analysis and generalized differentiation allows us to derive necessary optimality conditions for discrete-time problems and then pass to the limit with respect to the vanishing discretization step. In this way we derive nondegenerate necessary optimality conditions for the original sweeping control problems expressed in terms of given local minimizers and problem data. Some efficient applications of the obtained results to practical control problems of the sweeping type are also discussed in the talk.

Rate-independent unilateral evolutions for Ambrosio-Tortorelli functionals and applications

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Keywords : BV-evolution, alternate minimization, brittle fracture

We consider quasi-static evolutions for a phase-field Ambrosio-Tortorelli energy. We employ time discretization and at each time incremental problem we use an alternate minimization scheme, in which the (time) updated configuration is found by an iterative procedure converging to an equilibrium configuration of the system. Alternate schemes take full advantage of the fact that the energy is separately quadratic, even if globally non-convex. We consider in particular unilateral or monotonicity constraints for the phase-field variable, which models irreversibility in the applications to fracture and damage. Recasting the above alternate minimization problems as gradient flows, with respect to a suitable family of intrinsic state depending norms, allows to characterize the time continuous limit in terms of a (parametrized) BV-evolution, i.e. by equilibrium and energy identity. It is well known that BV-evolutions for rate-independent problems features both stable (or continuity) regimes and unstable (or discontinuity) regimes. In the stable regime the limit evolution is an equilibrium point of the energy; mechanically the phase-field variable enjoys a suitable form of Griffith's criterion and the monotonicity constraint turns out to be thermodynamically consistent with the dissipated energy. In the unstable regime the limit evolution is instead a sort of (parametrized) “normalized gradient flow" with respect to the state depending intrinsic norms; mechanically, it provides a Kelvin-Voigt visco-elastic flow in the displacement variable.

A similar result holds also generating BV-evolutions by vanishing viscosity, employing for instance a unilateral L^2-gradient flow for the phase field variable, which in turn can be generated by an alternate minimization scheme.

Sweeping Process and Optimal Control

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Keywords : Optimal Control, Differential Inclusion, Sweeping Process.

In this talk, we address some recent advances in Dynamic Optimization for the Controlled Sweeping Process (also known in the literature as Moreau's process)

$$(\star) \qquad \dot{x} \in -N_{C(t)}(x) + f(t,x,u), \qquad u \in U$$

where $C(.)$ is a Lipschitz continuous set-valued mapping. Such a framework is a general way to model optimal control problems with state constraints depending on time, but it also arises in many other applications like crowd motion, electric circuit, mechanical system modeling and soft-robotic. In the first part of the talk we present a minimum time problem related to $(\star)$ and we characterize the minimum time function $T(t,x)$ as the continuous solution of a set of Hamilton-Jacobi inequalities; in the second part we discuss some recent advances for what concerns the necessary conditions for a general Mayer problem. In both cases, the main difficulty is due to the presence of the normal cone $N_{C(t)}(x)$ on the right hand side of $(\star)$, which is a not Lipschitz continuous mapping with respect to x and contains implicitly the state constraint $x(t) \in C(t)$.

BV-norm continuity of sweeping processes driven by a set with constant shape

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Keywords : Sweeping processes, play operator, rate independence

A relevant class of sweeping processes (cf. [1]) is represented by those processes driven by a set $\mathcal{C}$ with constant shape, namely $\mathcal{C}(t) = u(t) - \mathcal{Z}$, where u is a mapping with bounded variation with values in a Hilbert space $\mathcal{H}$, and $\mathcal{Z}$ is a closed convex subset of $\mathcal{H}$. The solution operator of this important special sweeping process, also known as *play operator*, has a fundamental role in elastoplasticity (see [3]) and was already described by J.J. Moreau in one of his seminal papers in the early seventies (cf. [2]). In this talk we prove that the play operator is continuous with respect to the BV-norm with no regularity assumptions on $\mathcal{Z}$ and no restriction on the space $\mathcal{H}$. The proof of such result (cf. [6]) is based on a recent reparametrization technique for convex-valued curves introduced in [4] and [5].

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Optimal Control Problems with Mixed Constraints: Special Applications

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Keywords : some keywords

In this talk we consider various optimal control problems that can be reformulated as mixed constrained problems. We focus on necessary optimality conditions for such problems discussing the role of regularity conditions on mixed constraints. We give an overview on how necessary conditions for mixed constraints problems are used to for implicit control problems, problems involving differential algebraic problems and problems with order one state constraints. We then present some interesting problems that can be reformulated as problems with irregular mixed constraints. Those include some special cases of sweeping problems.

Evolution PDEs under density constraints

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Keywords : optimal transport, crowd motion, degenerate diffusion, sweeping process

I will present a survey on a general model, mainly developed with Bertrand Maury, but also with and by other collaborators, for crowd motion in a macroscopic setting. From the point of view of crowd motion, the model involves hard congestion effects and is translated into a PDE with an upper bound on the density. I will insist in particular on the key ideas for approximating solutions after discretizing in time, which allows both to prove existence and to obtain efficient bumerical methods. This approximation requires the use of tools from optimal transport theory and can, under some assumptions, fits the general theory of gradient flows in the metric space of probability densities endowed with the Wasserstein distance. This general model stemmed from a corresponding microscopic model introduced by Maury and Venel, which could be expressed as a high-dimensional sweeping process, or differential inclusion. Here another connection with the sweeping process will appear in the case where we consider a moving domain.

Inequality Problems, Sweeping Precesses and Optimal Control in Contact Mechanics

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Keywords : contact problem, variational-hemivariational inequality, sweeping process, optimal pair, weak solution, convergence results.

Contact between deformable bodies abounds in industry and everyday life. A few simple examples are brake pads in contact with wheels, tires on roads, and pistons with skirts. Because of the importance of contact processes in structural and mechanical systems, considerable effort has been put into their modeling, analysis and numerical simulations. The literature in the field is extensive, as it results from [1] and the references therein.

Our aim in this lecture is to show that the variational analysis of contact problems leads to new and non standard mathematical formulations. To this end, we consider two mathematical models which describe the contact of viscoelastic and elastic materials, respectively.

For the viscoelastic model we derive three variational formulations; the first one is in a form of a sweeping process for the diaspacement field, the second one is in a form of a history-dependent variational inequality for the velocity field and the third one is in a form of a time-dependent variational inequality for the stress field. For each variational formulation we provide existence and uniqueness results of the solution, by using different assumptions and arguments. Then, we compare these results and formulate some open problems. We also study the link between the corresponding solutions and provide equivalence results, together with their mechanical interpretations.

For the elastic model we derive a variational formulation which is in a form of a variational-hemivariational inequality for the displacement field. Then, we prove its unique weak solvability. We also formulate two optimal control problems for which we prove the existence of optimal pairs, together with some convergence results.

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Some Remarks on State Constraints and Mixed Constraints

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Keywords : variational inequalities, interiority assumptions, implicit parametrization

In the setting of state constrained control problems, we discuss an approximation technique involving variational inequalities. The constraints are automatically satisfied in this procedure. For control problems with mixed constraints, a relaxation of classical interiority assumptions is presented together with a recent approach based on implicit parametrization results.

Positive minimal time for the control of state constrained dynamical systems

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Keywords : Control of state constrained dynamical systems

Given a finite-dimensional linear control system satisfying the Kalman condition, or given the heat equation in a bounded domain with internal or boundary control, if there are no state or control constraints, then controllability can be achieved in arbitrarily small time.

Now, we add some state constraints and we investigate the question of knowing if one can still pass from any initial condition to any final target in arbitrarily small time. Surprisingly, this question has received little attention, even in the simplest case of finite-dimensional linear control systems. We prove that, in this very simple case, if the initial state and target state are steady states, then controllability can be achieved in time large enough, and we prove that in general there exists a positive minimal time, even for unilateral linear state constraints (like nonnegativity of the state).

In the infinite dimensional setting, we focus on the heat equation with homogeneous Dirichlet boundary conditions, which is a well known model that preserves nonnegativity. Besides, due to infinite velocity propagation, without any constraint, the heat equation is null-controllable within arbitrary small time, with controls supported in any arbitrarily open subset of the domain (or its boundary) where heat diffuses.

Considering a positive initial condition and a positive target steady-state, we prove that controllability can be achieved, keeping a nonnegative state all along the trajectory, in time large enough, and that there is a positive (i.e., nonzero) minimal time. In other words, in spite of infinite velocity propagation, realizing controllability under an unilateral nonnegativity state constraint requires a positive minimal time.

Similar results are obtained for unilateral control constraints, and also for other models, including more general parabolic models, various boundary conditions, various (internal or boundary) controls.

Differential inclusions and applications

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Keywords : sweeping process, crowd motion

My talk will deal with differential inclusions. Such evolution problems appear when the state-variable is submitted to some constraints and therefore has to stay in an admissible set. Especially we will be interested in the study of sweeping processes and of second order differential inclusions involving proximal normal cones. We propose to detail some results about these different problems by pointing out the geometrical assumptions regarding the admissible sets. Furthermore we will consider some applications in crowd motion modelling and in granular media.

Optimal Control of Linear Complementarity Systems

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Keywords : LQ Optimal Control, Linear Complementarity Systems, MPCC, Mixed Constraints

This presentation will focus on the optimal control of a specific kind of constrained systems, called Linear Complementarity Systems (LCS). Namely, we want to find the control $u(\cdot)$ solving the following problem :

$$\begin{aligned} &\min \int_0^T \left(x(t)^\intercal Q x(t) + u(t)^\intercal U u(t)\right) dt \\ &\text{s.t. } \dot{x}(t) = Ax(t) + Bv(t) + Eu(t) \qquad \text{a.e. on } [0,T] \\ &\qquad 0 \leq v(t) \perp Cx(t) + Dv(t) + Eu(t) \geq 0 \quad \text{a.e. on } [0,T] \end{aligned} \tag{0.5}$$

where

- A, B, E, C, D, E, Q and U are matrices of according dimensions
- the inequalities are understood component-wise, and $a \perp b$ means $a^\intercal b = 0$.

When we deal with this kind of optimisation problems in the finite-dimensional case, this problem is called Mathematical Program with Complementarity Conditions (MPCC). They are known to be hardly tractable, due to the violation at every feasible point of the usual Constraints Qualification (CQ). Even if v can be expressed uniquely for some fixed x and u, the corresponding dynamics becomes non-differentiable, and thus we can not apply usual techniques of first-order conditions. This talk will mainly show where these systems can be found, what mathematical framework is needed in order to tackle this problem, and some analytical and numerical results.

Galerkin-Like method for generalized perturbed sweeping process with nonregular sets

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Keywords : Sweeping process, Galerkin-like method, Positively α-far sets.

In this talk, we describe the Galerkin-Like method to solve differential inclusions in separable Hilbert spaces. This method consists in approaching the original problem by projecting the state into a n-dimensional Hilbert space, but not the velocity. We prove that approached problems always has a solution and that, under some compactness conditions, they converge strongly pointwisely (up to a subsequence) to a solution of the original differential inclusion.

As an application of this method, we provide existence of solutions for the Generalized Sweeping Process:

$$\begin{cases} -\dot{u}(t) = Bv(t) \\ -\dot{v}(t) \in N\left(C(t,u(t),v(t)); v(t)\right) + F(t,u(t),v(t)) + Au(t) \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0,u_0,v_0), \end{cases}$$

where $A\colon U \to V$ and $B\colon V \to U$ are two bounded linear operators, $N(S;\cdot)$ denotes the Clarke normal cone to a closed set $S \subseteq V$ and $F\colon [T_0,T] \times U \times V \rightrightarrows V$ is a set-valued map with nonempty closed and convex valued satisfying appropriate conditions.

The Generalized Sweeping Process includes the perturbed state-dependent, the Moreau's sweeping process and the perturbed second-order sweeping process, among others, for which we obtain general existence results.

A Unified Approach to State Constrained Optimal Control, Based on Distance Estimates

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Estimates on the distance of a nominal state trajectory from the set of state trajectories that are confined to a closed set have an important unifying role in optimal control theory. They can be used to establish non-degeneracy of optimality conditions such as the Pontryagin Maximum Principle, to show that the value function describing the sensitivity of the minimum cost to changes of the initial condition is characterized as a unique generalized solution to the Hamilton Jacobi equation, and for numerous other purposes. We discuss the validity of various presumed distance estimates and their implications, recent counter-examples illustrating some unexpected pathologies and pose some open questions.

Derivatives of sweeping processes as continuous products of matrices

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Keywords : sweeping process, projection matrix, left convergent products, Moore-Smith convergence

The sweeper $Z(t)$ transfers $x(0) \in Z(0)$ to $x(T) \in Z(T)$ and this map is 1-Lipschitz, hence, a.e. differentiable. In some cases, the value of the Jacobian matrix at a given $x(0)$ can be found as a special infinite product of projection matrices.

Here at each step the current finite product of matrices is extended to a longer product by means of insertion of a new matrix from a given family Σ into an arbitrary position within the product, the front and the rear ends included. We say that a finite family Σ of matrices has CP-property if this procedure always converges to some matrix called the continuous product that depends, in general, on the order of insertions.

We study finite classes of matrices that possess both the left convergence property and the right convergence property, that is, whose infinite products in a conventional sense converge in both directions. The main theorem says that these classes are exactly the classes of matrices that have the CP-property.

There are extensions to sweeping processes with oblique reflection.

Optimal control of a rate-independent evolution equation via viscous regularization

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Keywords : rate-independent system, optimality conditions

We study the optimal control of a rate-independent system that is driven by a convex quadratic energy. Since the associated solution mapping is non-smooth, the analysis of such control problems is challenging. In order to derive optimality conditions, we study the regularization of the problem via a smoothing of the dissipation potential and via the addition of some viscosity. The resulting regularized optimal control problem is analyzed. By driving the regularization parameter to zero, we obtain a necessary optimality condition for the original, non-smooth problem.

The method of characteristics in Fully Convex Control problems with state constraints.

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Keywords : Fully convex control, state constraints, method of characteristics

We will present the duality relationships displayed by value functions in Fully Convex Control (FCC) that have state constraints. The method of characteristics will be shown to extend to this setting.

Value Function and Optimal Trajectories for some State-Constrained Control Problems

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Keywords : State constraints, value function, trajectory reconstruction

In this talk, we shall consider some deterministic optimal control problems with state constraints and non-linear dynamics. We shall first review the regularity of the value function and its characterization as unique solution of a Hamilton-Jacobi equation. Then, we shall focus on an idea introduced in [3] that allows to describe the epigraph of the value function by an auxiliary optimal control problem free of state constraints, and for which the value function is Lipschitz continuous and can be characterized, without any controllability requirement, as the unique viscosity solution of a Hamilton-Jacobi equation. This idea leads to several trajectory reconstruction procedures for which convergence results are also investigated. An application to a five-state aircraft abort landing problem is then considered, for which several numerical simulations are performed to analyse the relevance of the theoretical approach.

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Controllability properties of dynamical systems with hysteresis

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Keywords : Hysteresis, controllability, rate independece

In the framework of dynamical control systems in which the controls are the components of a magnetic field, it is natural to consider a hysteresis phenomenon which can be modeled inserting in the equation a so-called hysteresis operator. This kind of operators are non linear and non differentiable, moreover the dependence on the past history prevents the use of local techniques, and hence the application of classical tools in geometric control theory is not immediate to get controllability results.

More precisely we focused on affine control systems, of which the magnetic microswimmer model is an example, and we analyzed how to introduce an hysteresis operator. We introduced in the system the *Play operator* in two different ways. On one hand we applied it on the controls and prove that if the system without hysteresis is controllable, we are always able to fine a sequence of continuous controls that makes the system with hysteresis controllable. On the other hand we can introduce the hysteresis on the state variables and we are able to recover some approximate controllability results for a class of affine control systems that have specific characteristics.